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Proceeding to the case of the equations

$$ax+b+\sqrt{(cx^2+d)}=0\ldots\ldots(4),$$
$$ax+b-\sqrt{(cx^2+d)}=0.,\ldots.(5).$$

Put $ax+b=y$, then (4) becomes

$$-\sqrt{c\left(\frac{y-b}{a}\right)^2+d}=+y\ldots\ldots(6),$$

and (5) becomes

$$\sqrt{c\left(\frac{y-b}{a}\right)^2+d}=+y\ldots\ldots(7).$$

If (6) is to be satisfied by a value of y, this value cannot be positive since $\sqrt{[f(y)]}+1=0$ is an impossible relation as was proved in the case of (2). The product of (6) and (7) becomes, after simplifying,

$$(c-a^2)y^2+2bcy+a^2d+b^2c=0\ldots\ldots(8).$$

Now it is evident that if (8) gives a positive value for y then (6) is not satisfied but (7) is; if (8) gives a negative root the reverse is the case.

In this way the following criteria are deduced, assuming that a^2d+b^2c-cd is positive. Putting

$$A=\frac{2bc}{c-a^2},\text{ and }B=\frac{a^2d+b^2c}{c-a^2}.$$

If $A>0$ and $B>0$, then (4) has no roots and (5) has two roots.

If $A>0$ and $B<0$, or $A<0$ and $B<0$, then (4) has one root and (5) has one root.

If $A<0$ and $B>0$, then (4) has two roots and (5) has no roots..

This solves the problem as originally stated. In an analogous way one could deduce criteria for the solubility of irrational equations of any type whatever.

This problem was also discussed by J. M. Boorman, who arrived at the conclusion that there is no condition for the equation to have no roots. He maintains that Newton's (his) theorem "Every algebraic equation has as many roots as it has dimensions," holds good. But this is not true. D'Alembert's theorem does not apply in this case at all since the equation is not a rational algebraic equation. It may not be out of place to state here D'Alembert's theorem in full: "Every rational, integral, algebreic equation whose coefficients are imaginary quantities, or, in special cases, real quantities, has at least one root." When this theorem is proved it follows as a corollary that there are as many roots as there are units in the number expressing the degree of the equation. Ed.

132.. Proposed by J. SCHEFFER. A. M., Hagerstown, Md.

Solve $2^x+3^y=4$; $5^x+6^y=7$.

Solution by H. C. WHITAKER, C. E., Ph. D., Manual Training School, Philadelphia, Pa.; MARCUS BAKER, U. S. Geological Survey, Washington, D. C.; and G. B. M. ZERR, A. M., Ph. D., The Temple College, Philadelphia, Pa.

Plotting the curves represented by the two equations, they are seen to intersect at (0, 1) and also in the neighborhood of (.6, .8).

$$\frac{\log(7-6^y)}{\log 5}-\frac{\log(4-3^y)}{\log 2}=0.$$

By Double Position the roots near .6, .8 are found to be $x=.565585$, $y=.841307$; these with the values $x=0$, $y=1$ seem to be the only real roots.

Also solved by *JOHN G. KELLER*, State Normal School, Albany, N. Y.
The Proposer sent results only, to-wit: $x=.56556312$, $y=.8413092$.

GEOMETRY.

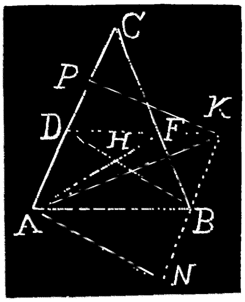
123, **Proposed by P. C. CULLEN, Indianola, Iowa.**

If the bisectors of the base angles of a triangle are equal the triangle is isosceles.

I. Direct Demonstration by G. I. HOPKINS, A. M., Professor of Physics and Astronomy, High School, Manchester, N. H.

CONSTRUCTION. Make $\angle BDK=\angle FAB$, and draw through B the line NK, making $\angle DBK=\angle AFB$. Draw the perpendiculars KP and AN, and join A and K.

DEMONSTRATION. Since angles FAB and AFB are two angles of a triangle, DK and NK will meet.



$\therefore$ triangles DBK and AFB are equal.

$\therefore$ $DK=AB$ and $BK=BF$.

$\angle AHB=\angle ADH+\angle DAH=\angle ADH+\angle BDK$.

$\therefore$ $\angle AHB=\angle ADK$. Also $\angle AHB=\angle HBF+\angle HFB=\angle HBA+\angle HBK$.

$\therefore$ $\angle AHB=\angle ABK$. $\therefore$ $\angle ADK=\angle ABK$.

$\therefore$ $\angle KDP=\angle ABN$. $\therefore$ $\triangle DPK=\triangle ABN$.

$\therefore$ $AN=KP$, and $NB=DP$.

$\therefore$ $\triangle APK=\triangle AKN$.

$\therefore$ $AP=NK$, and $\therefore$ $AD=BK$. $\therefore$ $AB=FB$. $\therefore$ $\triangle ADK=\triangle AFB$.

$\therefore$ $\angle ABD=\angle FAB$. $\therefore$ $\angle DAB=\angle ABF$.

$\therefore$ $AC=BC$, and the triangle ABC is isosceles. Q. E. D.

II. Demonstration by L. E. DICKSON, Ph. D., Assistant Professor of Mathematics, University of Chicago, Chicago, Ill.

Let d_a be the length of the bisector of the angle A opposite to side a. Expressing the area of the given triangle and the areas of the triangles into which it is divided by the bisector of angle A, we get

$$\tfrac{1}{2}bc\sin A=\tfrac{1}{2}bd_a\sin\tfrac{1}{2}A+\tfrac{1}{2}cd_a\sin\tfrac{1}{2}A.$$